

# ***Harrod Domar Growth Model***

BA Sem VI

Development Economics

Code ECB-604

Presented by

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- Ever since the end of Second World War, interest in the problems of economic growth has led economists to formulate growth models of different types.
- These models deal with and lay emphasis on the various aspects of growth of the developed economies.
- They constitute in a way alternative stylized pictures of an expanding economy.

- The Harrod–Domar model is a Keynesian model of economic growth.
- It is used in development economics to explain an economy's growth rate in terms of the level of saving and of capital.
- It suggests that there is no natural reason for an economy to have balanced growth.
- The model was developed independently by R. F. Harrod in 1939 and Evsey Domar in 1946 although a similar model had been proposed by Gustav Cassel in 1924.
- The Harrod–Domar model may be regarded as the precursor to the exogenous growth model.

## Assumptions

(i) A full-employment level of income already exists.

(ii) There is no government interference in the functioning of the economy.

(iii) The model is based on the assumption of “closed economy.” In other words, government restrictions on trade and the complications caused by international trade are ruled out.

(iv) There are no lags in adjustment of variables i.e., the economic variables such as savings, investment, income, expenditure adjust themselves completely within the same period of time.

as functional equality between saving and investment.

(v) The average propensity to save (APS) and marginal propensity to save (MPS) are equal to each other.  $APS = MPS$  or written in symbols,

$$S/Y = \Delta S / \Delta Y$$

(vi) Both propensity to save and “capital coefficient” (i.e., capital-output ratio) are given constant.

This amounts to assuming that the law of constant returns operates in the economy because of fixity of the capital-output ratio.

(vii) Income, investment, savings are all defined in the net sense, i.e., they are considered over and above the depreciation. Thus, depreciation rates are not included in these variables.

(viii) Saving and investment are equal in ex-ante as well as in ex-post sense i.e., there is accounting as well.

## **Harrod's growth model raised three issues:**

- (i) How can steady growth be achieved for an economy with a fixed (capital- output ratio) (capital-coefficient) and a fixed saving-income ratio?
  
- (ii) How can the steady growth rate be maintained? Or what are the conditions for maintaining steady uninterrupted growth?
  
- (iii) How do the natural factors put a ceiling on the growth rate of the economy?

In order to discuss these issues, Harrod had adopted three different concepts of growth rates:

(i) the actual growth rate,  $G$ ,

(ii) the warranted growth rate,  $G_w$

(iii) the natural growth rate,  $G_n$ .

- The Actual Growth Rate is the growth rate determined by the actual rate of savings and investment in the country. In other words, it can be defined as the ratio of change in income ( $\Delta Y$ ) to the total income ( $Y$ ) in the given period. If actual growth rate is denoted by  $G$ , then
- $G = \Delta Y / Y$
- The actual growth rate ( $G$ ) is determined by saving-income ratio and capital-output ratio. Both the factors have been taken as fixed in the given period. The relationship between the actual growth rate and its determinants was expressed as:
- $G C = s \dots (1)$
- where  $G$  is the actual rate of growth,  $C$  represents the capital-output ratio  $\Delta K / \Delta Y$  and  $s$  refers to the saving-income ratio  $\Delta S / \Delta Y$ . This relation states the simple truism that saving and investment (in the ex- post sense) are equal in equilibrium.



- The actual growth rate (G) is determined by **saving-income ratio** and **capital- output ratio**, taken as fixed in the given period.

The relationship between the **actual growth rate** and its **determinants** was expressed as:

$$GC = s \dots(1)$$

- where G is the actual rate of growth,
- C represents the capital-output ratio  $\Delta K/\Delta Y$  and
- s refers to the saving-income ratio  $\Delta S/\Delta Y$ .
- This relation states the simple truism that saving and investment (in the ex- post sense) are equal in equilibrium.

Since

$$G = \frac{\Delta Y}{Y}$$

$$C = \frac{\Delta K}{\Delta Y} = \frac{I}{\Delta Y} \quad [ \because \Delta K = I ]$$

Because

$$s = \frac{S}{Y}$$

Substituting the value of  $G$ ,  $C$ , and  $s$  in equation (1), we get

$$\frac{\Delta Y}{Y} \times \frac{I}{\Delta Y} = \frac{S}{Y}$$

or

$$\frac{I}{Y} = \frac{S}{Y}$$

or

$$I = S$$

- This relation explains that the condition for achieving the steady state growth is that ex-post savings must be equal to ex-post investment.

# Warranted growth

- It refers to that growth rate of the economy when it is working at full capacity.
- also known as **Full-capacity growth rate**.  
denoted by  $G_w$ .
- is the rate of income growth required for full utilisation of a growing stock of capital, so that entrepreneurs would be satisfied with the amount of investment actually made.
- $G_w$  is determined by **capital-output ratio** and **saving- income ratio**.
- The relationship can be expressed as

$$G_w C_r = s$$

- where  $C_r$  shows the needed  $C$  to maintain the warranted growth rate and  $s$  is the saving-income ratio.
- Harrod- An economy can achieve steady growth when

$$G = G_w \text{ and } C = C_r$$

- This condition states, firstly, that actual growth rate must be equal to the warranted growth rate.
- Secondly, the capital-output ratio needed to achieve  $G$  must be equal to the required capital-output ratio in order to maintain  $G_w$ , given the saving co-efficient ( $s$ ).
- This amounts to saying that **actual investment** must be equal to the **expected investment** at the given saving rate.

## Instability of Growth:

- As stated above that the steady-state growth of the economy requires an equality between  $G$  and  $G_w$  on the one hand and  $C$  and  $C_r$  on the other.
- In a free-enterprise economy, these equilibrium conditions would be satisfied only rarely, if at all.
- Therefore, Harrod analysed the situations when these conditions are not satisfied.

(i) If  $G > G_w^*$   
then  $C < C_r$

(ii) If  $G < G_w$   
then  $C > C_r$

When  $G$  is greater than  $G_w$ .

$$G > G_w$$

- Under this situation, the growth rate of income being greater than the growth rate of output,
- the demand for output (because of the higher level of income) would exceed the supply of output (because of the lower level of output) and the economy would experience inflation.
- This can be explained in another way too when

$$C < C_r$$

Under this situation, the actual amount of capital falls short of the required amount of capital.



- This would lead to deficiency of capital, which would, in turn, adversely affect the volume of goods to be produced.
- Fall in the level of output would result in scarcity of goods and hence inflation.
- This, under this situation the economy will find itself in the quagmire of inflation.

when  $G$  is less than  $G_w$ ,

$$G < G_w$$

- the growth rate of income would be less than the growth rate of output.
- In this situation, there would be **excessive goods for sale**, but the income would not be sufficient to purchase those goods.
- In Keynesian terminology, there would be deficiency of demand and consequently the economy would face the **problem of deflation**.
- This situation can also be explained when  $C$  is greater than  $C_r$ .

- Here the actual amount of capital would be larger than the required amount of capital for investment.
- The larger amount of capital available for investment would dampen the marginal efficiency of capital in the long period.
- Secular decline in the **marginal efficiency of capital** would lead to chronic depression and unemployment.
- This is the state of **secular stagnation**.

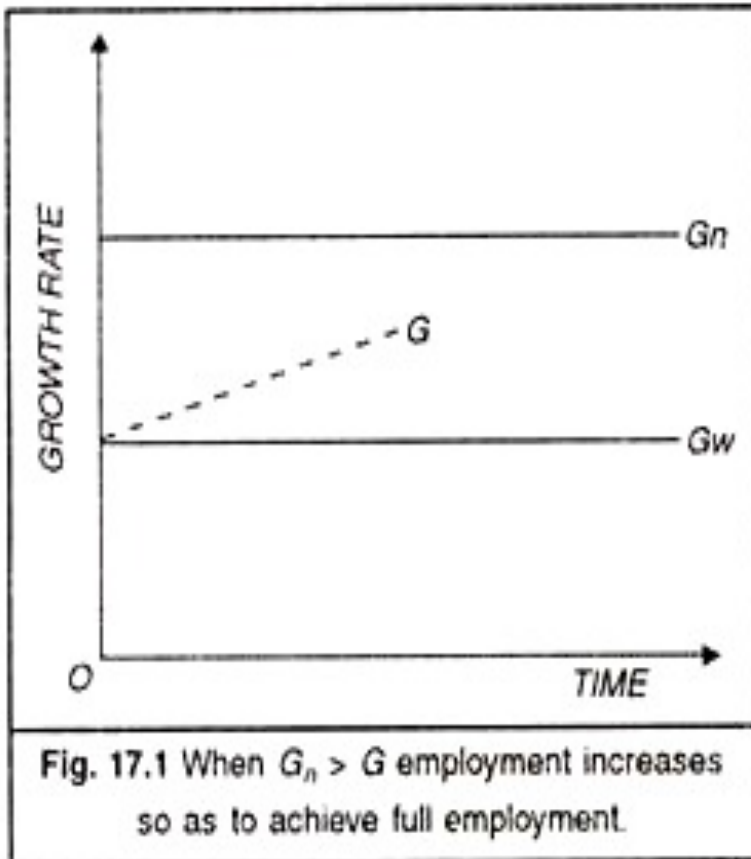
- From the above analysis, it can be concluded that steady growth implies a balance between  $G$  and  $G_w$ .
- In a free-enterprise economy, it is difficult to strike a balance between  $G$  and  $G_w$  as the two are determined by altogether different sets of factors.
- Since a slight deviation of  $G$  from  $G_w$  leads the economy away and further away from the steady-state growth path, it is called 'knife-edge' equilibrium.

# Natural growth Rate $G_n$

- determined by natural conditions such as **labour force, natural resources, capital equipment, technical knowledge** etc.
- These factors place a limit beyond which expansion of output is not feasible.
- This limit is called **Full-Employment Ceiling**.
- This upper limit may change as the production factors grow, or as technological progress takes place.
- Thus, the **natural growth rate is the maximum growth rate which an economy can achieve** with its available natural resources.
- The third fundamental relation in Harrod's model showing the determinants of natural growth rate is

$$G_n C_r \text{ is either } = \text{ or } \neq s$$

# Interaction of $G$ , $G_w$ and $G_n$ :



- If  $G_n$  exceeds  $G_w$ , ( $G > G_w$ )
- $G_n$  would also exceed  $G_w$  for most of the time and there would be a tendency in the economy for cumulative boom and full employment.
- Such a situation will create an **inflationary trend**.
- To check this trend, savings become desirable because these would enable the economy to have a high level of employment without **inflationary pressures**.

# Interaction of $G$ , $G_w$ and $G_n$ :

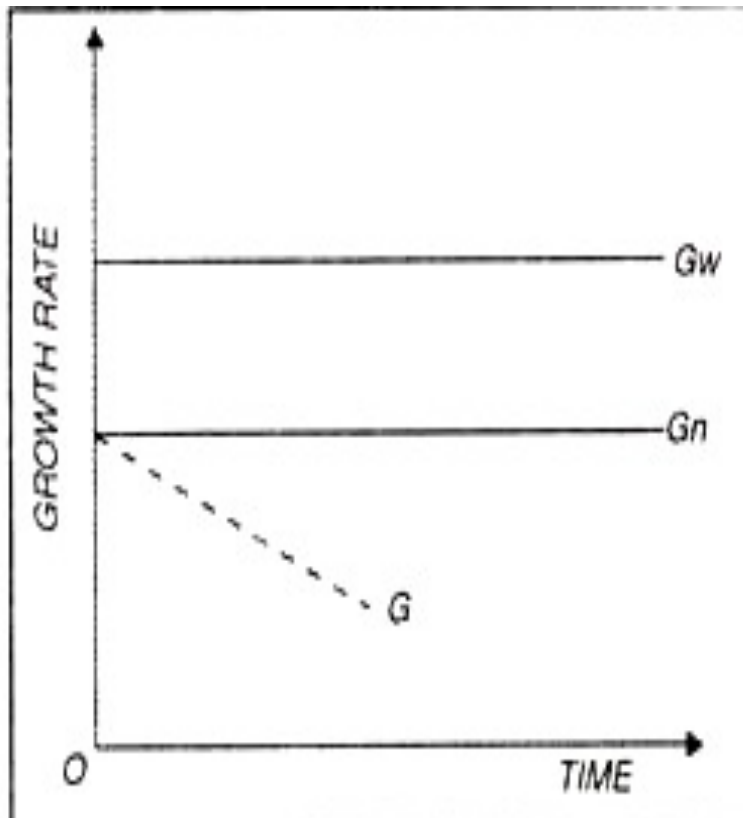


Fig. 17.2 When  $G_n > G_w$  there is growth of unused capacity of capital and employment continues to fall till  $G_w = G_n$

- If  $G_w > G$ ,
- $G$  must be below  $G_n$ ,
- for most of the time and there would be a tendency for cumulative **recession** resulting in **unemployment**

# The Domar Model:

- The main growth model of Domar bears a certain resemblance to the model of Harrod.
- In fact, Harrod regarded Domar's formulation as a rediscovery of his own version after a gap of seven years.



# 1. Investment has two effects

- (a) An income-generating effect and
  - (b) Productivity effect by creating capacity.
- 
- The keynes' Analysis which is short-run analysis ignored the second effect.

2. Unemployment of labour generally attracts attention and one feels sympathy for the jobless, but **unemployment of capital** attracts little attention.

It should be understood that unemployment of capital inhibits investment and hence reduces income.

Reduction of income brings about deficiency in demand and hence unemployment.

Thus the Keynesian concept of unemployment misses the root cause of the problem.

**Domar wanted to analyse the genesis of unemployment in a wider sense.** He explained following relations:

1. Income is determined by investment through multiplier. For simplicity saving-income ratio ( $s$ ) is assumed constant. This implies that

$$Y(t) = I(t)/s$$

where  $Y$  is the output,  $I$  is the actual investment and  $s$  is saving-income ratio (saving propensity) and  $(t)$  shows the time period.

2. Productive capacity is created by investment to the extent of the potential (social) average productivity of investment denoted by  $\sigma$ .

For simplicity, this is also assumed to be constant. In notation form the relation can be written as

$$Y_{(t)} - Y_{(t-1)} = I_{(t)} / \alpha$$

Where  $Y_{(t)} - Y_{(t-1)}$  refers to change in Productive Capacity in t period of time.

- $\alpha$  is the **marginal capital-output ratio** which is the reciprocal of “potential social average investment productivity” ( $\alpha = 1/\sigma$ ).
- Therefore, Equation (2) can also be expressed as
$$\Delta Y_t = \sigma I_t$$
- This equation shows that the change in productive capacity is the product of capital productivity ( $\sigma$ ) and investment.
- Hence, it reveals the productivity effect.

3) Investment is induced by output growth together with entrepreneur confidence.

4) Employment depends upon the 'utilization ratio' expressed as the ratio between actual output and productive capacity.

5) Past and present investment can greater productive capacity at a given ratio.

## Statement of the Model

- Domar model is based on the dual character of investment: **one, investment increases productive capacity**, and **two, investment generates income**.
- The two sides of investment provide solution for steady growth. The following symbols are used in DM.
- $Y_d$  = Level of **national income** or level of **effective demand** at full employment (demand side)
- $Y_s$  = Level of **productive capacity or supply** at full employment level (supply side)
- $K$  = real capital
- $I$  = net investment, which implies change in stock of real capital, i.e.  $\Delta K$
- $\zeta$  = marginal propensity to save, which is the reciprocal of multiplier i.e., ( $m_{ps} = 1/\text{multiplier}$ )
- $\sigma$  = productivity of capital

We can make use of these notations to frame a set of equations that help formulate the DM.

The **demand side of investment** can be represented by an equation as follows:

$$Y_d = I/\alpha \dots\dots\dots (1)$$

This equation explains two things as follows:

- i) **The level of effective demand ( $Y_d$ ) is directly related to the level of investment(I). An increase in investment will result in an increase in effective demand, and vice versa.**
- ii) **The effective demand is inversely related to the marginal propensity to save ( $\alpha$ ). An increase in marginal propensity to save will decrease the level of effective demand and vice-versa.**

The **supply side of investment** can be represented by an equation as follows:

$$Y_s = \sigma k \dots\dots\dots (2)$$



Eq.(2) explains that supply of output at full employment ( $Y_s$ ) depends upon two factors, i.e., productive capacity of capital ( $\sigma$ ) and the amount of real capital ( $K$ ).

Any change in any of these factors will result in a corresponding change in the supply of output. For example, an increase in the productivity of capital will result in an increase in output, and vice-versa.

Likewise, an increase in the amount of real capital will lead to an increase in output, and vice-versa.

## Equilibrium:

In equilibrium, the demand and supply should balance. Therefore,

$$Y_d = Y_s \dots\dots\dots (3)$$

$$\text{or } I/\alpha = \sigma K$$

By cross multiplication,

$$I = \alpha \sigma K$$

This is the condition for steady growth. **Steady growth is possible** when: Investment equals the product of **saving-income ratio, capital productivity** and **capital stock**.

For this we have to give increment to the demand and supply conditions presented above.

The demand equation in its incremental form can be stated as follows:

$$\Delta Y_d = \Delta I / \alpha \dots\dots\dots (4)$$

Increments have been shown in the level of effective demand and investment because they are variables,

but increment has not been shown in  $\alpha$  because it is constant in terms of the assumptions employed.

The supply equation in its incremental form can be stated as follows:

$$\Delta Y_S = \sigma \Delta K \dots\dots\dots (5)$$

- Eq.(5) explains that change in the supply of output ( $\Delta Y_s$ ) would be equal to the product of change in real capital ( $\Delta K$ ), and the productivity of capital ( $\sigma$ ).
- The change in real capital is expressed as net investment. Therefore,  $\Delta K$  represented investment ( $I$ ).

Substituting  $I$  in place of  $\Delta K$  in eq.(5), we get.

$$\Delta Y_s = \sigma I \dots\dots\dots(6)$$

- The equilibrium between eq.(4) and eq.(6) provides us the **condition for maintaining the steady growth**. In equilibrium

$$\begin{aligned} \Delta Y_d &= \Delta Y_s \\ \text{or } \Delta I / \alpha &= \sigma I \end{aligned}$$

cross-multiplying , we get,

$$\Delta I / I = \sigma \alpha \dots\dots\dots(7)$$

Eq.(7) explains that the growth-rate of net investment  $\Delta I/I$  should be equal to the product of marginal propensity to save ( $\alpha$ ) and productivity of capital ( $\sigma$ ).

This equality must be maintained to ensure stable and steady growth.

## Path of Disequilibrium

$$(i) \text{ When } \frac{\Delta I}{I} \text{ or } \frac{\Delta Y}{Y} > \alpha \sigma \qquad (ii) \text{ When } \frac{\Delta I}{I} \text{ or } \frac{\Delta Y}{Y} < \alpha \sigma$$

Under the first situation, long-term inflation would appear in the economy because the higher growth rate of income will provide greater purchasing power to the people and the productive capacity ( $\sigma\alpha$ ) would not be able to cope with the increased level of income.

The first situation of disequilibrium will, therefore, create **inflation in the economy**.

The second situation, under which growth rate of income or investment is lagging behind the productive capacity, will result in **over production**.

The reduced growth rate of income will put a constraint on the purchasing power of the people, thereby **reducing the level of demand and resulting in over-production**.

This is the situation in which there would be **secular stagnation**.

(i) When  $\frac{\Delta I}{I}$  or  $\frac{\Delta Y}{Y} > \alpha \sigma$

(ii) When  $\frac{\Delta I}{I}$  or  $\frac{\Delta Y}{Y} < \alpha \sigma$



## 2.5.1 Similarities

The two models are similar in substance. Harrod's is domar's  $d$ . Harrod's warranted rate of growth ( $G_w$ ) is Domar's full employment rate of growth ( $d\sigma$ ). Harrod's  $G_w = s/C_r \equiv$  Domar's  $d\sigma$ ).

To prove it

$$d = \frac{S}{Y} \text{ or } S = dY \quad \dots\dots\dots(1)$$

$$\sigma = \frac{\Delta Y}{I} \text{ or } \Delta Y = I\sigma \quad \dots\dots\dots(2)$$

Since  $S=I$ , and substituting  $S$  for  $I$  in eq.(2), we have

$$\Delta Y = dY\sigma \quad [ \because S=dY ]$$

or  $\frac{\Delta Y}{Y} = d\sigma \quad \dots\dots\dots(3)$

$$\cdot \quad G_w = d\sigma \quad (\text{since } G_w = \frac{\Delta Y}{Y})$$

## Dissimilarities between the Domar Model and Harrod Model.

	<b>Parameter</b>	<b>Domar</b>	<b>Harrod</b>
1.	Long-run difficulty	"Under-investment sapping growth"	Labour shortage deflecting growth
2.	Position of labour input	Shortage of certain labour may trigger scrapping and the inhibition of investment; optional element	Determinant of natural rate of growth: key element
3.	Centrifugal force from equilibrium	Continuously undermined investment incentives	Unstable adjustment process
4.	Reason for fixed capital output ratio	Assumed for inconvenience	Due to fixed interest rate, low substitutability, etc.
5.	State of economy	Idle capacity prevalent	Labour unemployment common place

## *References:*

- 1 Harrod, Roy F. (1939). "An Essay in Dynamic Theory". The Economic Journal.
- 2 Domar, Evsey (1946). "Capital Expansion, Rate of Growth, and Employment". Econometrica.

***Thank You***